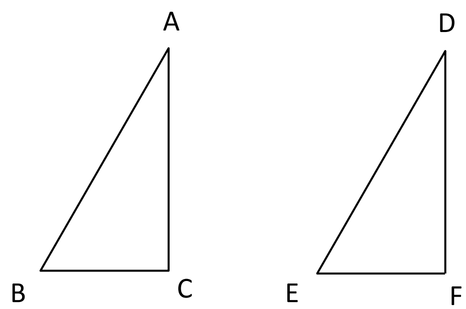
# Propositions 3, 5 and 25

## Triangle Congruency Propositions

In working with and understanding geometric figures it is very important to have a thorough knowledge of the properties of triangles. Most complex figures can be decomposed into a number of triangles whose separate properties can aid in understanding the properties of the whole figure.

There are three propositions that can be employed to prove that two or more triangles are congruent, or identical. Recall that each triangle can be thought of as consisting of 6 parts— 3 sides and 3 angles. Let's consider the two triangles below.




In either triangle it is enough to show that if 3 parts of one correspond to 3 parts of the other the triangles will be identical (with two exceptions). For example, if the side AB in the triangle ABC equals in length the side DE in the triangle DEF, and side AC in the first equals in length the side DF of the second, and the angle at A equals in measure the angle at D, then triangle ABC will exactly superimpose onto triangle DEF with a perfect fit. Side BC will exactly cover side EF, angle B will exactly equal angle E and angle C will exactly equal angle F. This proposition is typically known as the Side-Angle-Side Proposition or S.A.S. for short. The Side-Angle-Side Proposition is used in the next proposition to prove an important property of isosceles triangles.

Proposition 3: Triangles which have two sides and the angle contained by them equal are identical.

Also, it should be obvious without a lengthy, formal discussion that if the three sides of one triangle equal in length the three sides of the other, the triangles are identical, that is, if side AB equals side DE, side AC equals side DF and side BC equals side EF there can be no other outcome than perfect congruency between the two triangles. This proposition, numbered 5 in our sequence, is known as the Side-Side-Side Proposition, or S.S.S. for short.

Proposition 5: Triangles which have their three sides equal are identical.

Finally, if any two angles of one triangle equal corresponding angles in the other, and the sides contained between them are equal in length, the two triangles must be identical. For example, in the triangles above, if angle A equals angle D and angle B equals angle E, and the sides AB and DE are equal in length, then all the rest of the parts must coincide and the two triangles are identical. This is the Angle-Side-Angle Proposition (A.S.A.) and is numbered 25 in our sequence.

Proposition 25: Triangles which have two angles and the side which lies between them equal are identical.

The side length in question must lie between the two equal angles to guarantee congruency. While two sides and one angle, two angles and one side, or three sides equal can prove congruency of triangles, three angles equal does not mean the triangles are identical, that is, there is no Angle-Angle-Angle proposition of congruency. Obviously, two identical triangles will have all corresponding angles equal; however the converse is not true. Two triangles can have their corresponding angles equal but be of widely different size. For example, a triangle of side lengths 3, 4 and 5 units will have the same angles as a triangle of side lengths 6, 8 and 10 units. The triangles will be the same shape, but different size. Triangles that have the same shape but different size are called ***similar*** triangles.