

The Propositions of Euclidian Geometry

1. All radii of the same circle are equal.
2. The construction of the equilateral triangle by means of the vesica.
3. Triangles which have two sides and the angle contained by them equal are identical. (SAS)
4. In an isosceles triangle the angles at the base are equal.
5. Triangles which have all three sides equal are identical. (SSS)
6. To divide a straight line into two equal parts.
7. From a given point not on a straight line to draw a perpendicular to that line.
8. From a given point on a straight line to draw a perpendicular to that line.
9. The diameter of a circle divides the circumference into two equal parts.
10. A straight line which meets another straight line forms with it two angles which together are equal to two right angles.
11. A straight line drawn perpendicular to another straight line makes right angles with it.
12. If two straight lines intersect the vertical, or opposite, angles are equal.
13. If a line is perpendicular to one of two parallel lines, it is also perpendicular to the other.
14. If one line is perpendicular to two other lines, these two lines are parallel.
15. The opposite sides of a rectangle are parallel.
16. The opposite sides of a rectangle are equal.
17. A straight line falling upon parallel lines makes the alternate angles equal.
Corollary: In identical triangles the equal angles are always opposite equal sides.
18. If one straight line falling upon two other lines makes the alternate angles equal, these two lines are parallel.
19. If one line falls upon two parallel lines, it makes the interior angle equal to the exterior.

20. If one line falling upon two other lines makes the internal angle equal to the external, those two lines are parallel.
21. Through a given point to draw a parallel to a given line.
22. The three angles of a triangle taken together are equal to two right angles.
Corollary: If two angles of any triangle are known, the third is also known, for it is that which the other two together require to be equal to two right angles.
23. If two triangles have two angles equal, they have also the third angle equal.
24. The exterior angle of any triangle is equal to the two interior and opposite angles taken together.
25. Triangles which have two angles and the side which lies between them equal are identical. (ASA)
26. If two angles of a triangle are equal, the sides opposite to those angles are equal.
27. The opposite sides of a parallelogram are equal.
28. Parallelograms which are between the same parallels, and have the same base, \ are equal.
29. If a triangle and a parallelogram are upon the same base, and between the same parallels, the triangle is equal to half the parallelogram
30. Parallelograms which are between the same parallels, and have equal bases are equal.
31. Triangles which are between the same parallels, and have equal bases, are equal.
32. In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. (The Pythagorean Theorem)
33. To draw the circumference of a circle through three given points.

34. If the radius of a circle bisects a chord, it is perpendicular to that chord.

Corollary: The two radii intersecting the endpoints of the chord, form with the given radius two central angles which are also equal.

35. To find the center of a circle

36. To find the center of an arc of a circle.

37. If three equal lines meet in the same point within a circle, and are terminated at that point, they are radii of that circle.

38. If the radius of a circle is perpendicular to a chord, the radius bisects both the chord and the arc of the chord.

39. A straight line perpendicular to the extremity of a radius is a tangent to the circle.

Corollary: A perpendicular is the shortest line that can be drawn from any point to a given line.

40. If a straight line is drawn touching a circumference, a radius drawn to the point of contact will be perpendicular to the tangent.

41. The angle formed by a tangent and a chord is measured by half the arc of that chord.

42. An angle at the circumference of a circle is measured by half the arc which it contains (or, by which it is subtended.)

43. The angle at the center of a circle is double the angle at the circumference.

44. Upon a given line, to describe a segment of a circle containing a given angle.

45. In every triangle the greater side is opposite to the greater angle, and the greater angle to the greater side.

46. Two parallel chords intercept equal arcs.

47. If a tangent and a chord are parallel to each other, they intercept equal arcs.

48. The angle formed by the intersection of two chords is measured by half the arcs intercepted by the two chords.

49. The angle formed by two secants is measured by half the difference of the two intercepted arcs.

50. The angle formed by two tangents is measured by half the difference of the two intercepted arcs.

Corollary: By the same logic it can be demonstrated that the angle formed by a tangent and a secant meeting in the same point outside the circle, is measured by half the difference of the two intercepted arcs.

51. To raise a perpendicular at the end of a given line segment.

52. From any point outside of a circle to draw a tangent to that circle.

53. The surface area of a rectangle is equal to the product of its two sides.

54. The surface area of a triangle is equal to half the product of its altitude and its base.

55. To measure the surface of any rectilinear figure.

56. The area of a circle is equal to half the product of its radius and circumference.

57. To draw a triangle equal to a given circle.

58. The area of parallelograms which are between the same parallel lines are to one another as their bases.

59. Triangles which are between the same parallels are to one another as their bases.

60. If a line is drawn in a triangle parallel to one of its sides, it will cut the other two sides proportionately.

61. Equiangular triangles have their homologous (or corresponding) sides proportional.

62. Triangles which have their sides proportional are equiangular. (similar)

63. Triangles which have an angle in one equal to an angle in the other, and the sides adjacent to these angles proportional, are equiangular. (similar)
64. A straight line which bisects any angle of a triangle divides the side opposite to the bisected angle into two segments, which are proportional to the other two sides.
65. To find a fourth proportional to three given lines.
66. To find a third proportional to two given lines.
67. If four lines be proportional, the rectangle, or product of the extremes is equal to the rectangle or product of the means.
68. Four lines which have the rectangle or product of the extremes equal to the rectangle or product of the means are proportional.
69. If four lines are proportional, they are also proportional alternately.
70. If four lines are proportional, they will be proportional by composition. (or addition)
71. If four lines be proportional, they will also be proportional by division.
72. If three lines are proportional, the first is to the third as the square of the first is to the square of the second.
73. If two chords in a circle cut each other, the rectangle of the segments of one is equal to the rectangle of the segments of the other.
74. To find a mean proportional between two given lines.
75. The bases and altitudes of equal triangles are in reciprocal or inverse ratio.
76. Triangles which have the bases and altitudes in reciprocal or inverse ratio are equal.
77. Two secants drawn from the same point to a circle are in the inverse ratio of the parts which lie outside the circle.
78. The tangent to a circle is a mean proportional between the secant and the part of the secant which lies outside of the circle.
79. To divide a line in extreme and mean ratio: The Golden Mean.